Estimation Methods for Quality Factors of Inductors Fabricated in Silicon Integrated Circuit Process Technologies

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***Abstract*— By examining uses of quality (Q) factors for inductors in silicon integrated circuit design, new methods for estimating quality factors are proposed. These methods extract Q factors by numerically adding a capacitor in parallel to measured y_{11} data of an inductor, and by computing the frequency stability factor and 3-dB bandwidth at the resonant frequency of the resulting network. These parameters are then converted to effective quality factors using relationships for simple parallel RLC circuits. By sweeping the numerically added capacitance value, effective quality factors at varying frequencies are computed. These new techniques, in addition to being more relevant for circuit design, provide physically reasonable estimates all the way up to the self-resonant frequencies of inductors. At moderate to high frequencies, the commonly used Q definition $[-\mathrm{Im}(y_{11})/\mathrm{Re}(y_{11})]$ can significantly underestimate and can even give unreasonable results. Data obtained using the new methods suggest that quality factors remain high and integrated inductors remain useful all the way up to their self-resonant frequencies, contrary to the behavior obtained using $-\mathrm{Im}(y_{11})/\mathrm{Re}(y_{11})$. These indicate that the commonly used technique can lead to improper use and optimization of integrated inductors.**

***Index Terms*—Integrated inductors, quality factor, silicon IC's.**

I. INTRODUCTION

With the emergence of RF and microwave applications for silicon integrated circuits, integration of spiral inductors with reasonable characteristics has become an urgent need. In particular, quality (Q) factors of integrated inductors in silicon IC's are typically low, and the understanding and optimization of them have received intense attention [1]–[11]. Despite these efforts, the meaning of reported Q factors is still in a state of confusion. This paper examines existing methods for estimating Q factors and their limitations. To overcome these limitations, new methods, more relevant to circuit design, and useful up to the self-resonant frequencies of the inductors, are proposed. These methods extract quality factors as a function of frequency by numerically adding a capacitor with varying values in parallel to measured y_{11} data of an inductor, and by computing the frequency (phase) stability factors and 3-dB bandwidths at resonant frequencies of the resulting networks. The frequency stability factors and 3-dB bandwidths are then converted to effective quality factors using relationships for simple parallel RLC circuits. Quality factors obtained using these methods are compared to those obtained with the commonly used method based on $-\mathrm{Im}(y_{11})/\mathrm{Re}(y_{11})$ [1], [3]–[9], [11]. The results of this comparative study suggest that the commonly used method underestimates Q factors, and can lead to improper use and optimization of integrated inductors.

II. DEFINITIONS AND MEASUREMENTS OF QUALITY FACTORS

There are at least two widely used definitions for Q factors. The first and probably the most fundamental definition is based on the maximum energy storage and average power dissipation (P_{diss}) [12] which for simplicity, is referred as $Q_{E\mathrm{MAX}}$ in this paper

$$Q_{E\mathrm{MAX}} = \frac{\omega W_{\max}}{P_{diss}}. \tag{1}$$

The ω is the radian frequency, and $W_{\max}$ is the maximum total electrical and magnetic energies stored in the system. Unfortunately, as will be discussed later in this section, accurately estimating $Q_{E\mathrm{MAX}}$ is difficult.

The most widely used Q definition, which is referred as Q_{CONV} in this paper, is the ratio of a negative of the imaginary part of y_{11} and the real part of $y_{11}[-\mathrm{Im}(y_{11})/\mathrm{Re}(y_{11})]$ [1]. The y_{11} data are obtained by converting measured S-parameters of inductors. A common equivalent circuit for modeling integrated inductors in silicon IC processes [2] is shown in an inset of Fig. 1(a). The y_{11} data are utilized for the quality factor computations because they are the admittances seen looking into port 1 while port 2 is shorted to ground. This is a common configuration in which the inductors are used in amplifiers and oscillators.

Using simple network theory, it can be shown that

$$Q_{CONV} = -\left[\frac{\mathrm{Im}(y_{11})}{\mathrm{Re}(y_{11})}\right] = \frac{2\omega\left(|\overline{W}_m| - |\overline{W}_e|\right)}{P_{diss}} \tag{2}$$

where $|\overline{W}_m|$ and $|\overline{W}_e|$ are the average stored magnetic and electrical energies in the system [13]. When port 2 of the equivalent circuit model in Fig. 1(a) is shorted to ground, $|\overline{W}_m|$ and $|\overline{W}_e|$ are energies stored in the inductor L and capacitor C_{p1}, respectively. This Q definition involves the difference between the average stored magnetic and electrical energies rather than the maximum total energy storage. When the average magnetic energy storage is much greater than the electrical storage, this ratio approaches $Q_{E\mathrm{MAX}}$. For the general case as well as for silicon integrated inductors with typically large shunt capacitance to the substrate, thus, significant electrical energy storage, Q_{CONV} can deviate from $Q_{E\mathrm{MAX}}$ by a large amount.

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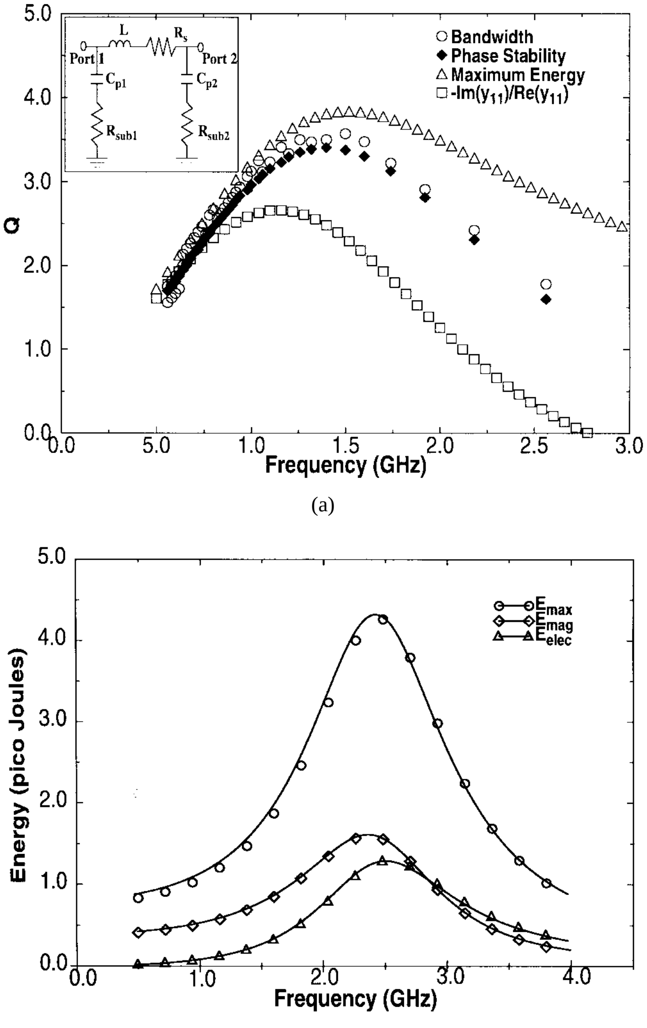


Fig. 1. (a) Quality factor versus frequency plots of a 15-nH inductor. An equivalent circuit commonly used to model integrated inductors in silicon IC's [2] is shown in an inset. The quality factors are extracted utilizing the commonly used measurement technique, one based on the maximum energy storage using the equivalent circuit model, and newly proposed methods. (b) The average magnetic and electrical energy storages, and the maximum energy storage as a function of frequency for the 15-nH inductor.

Fig. 1(a) shows Q versus frequency plots of a 15-nH inductor extracted with the conventional technique (Q_{CONV}), calculated based on the maximum energy storage using the equivalent circuit model ($Q_{E\mathrm{MAX}}$), and using the newly proposed methods discussed in Section III. Modeling parameters for the inductor as well as its geometry data are given in Table I. The inductors in Table I are fabricated on a 20-Ω-cm p-type substrate [6]. The Q_{CONV} and $Q_{E\mathrm{MAX}}$ are computed by applying a sine wave to port 1 of the equivalent circuit and by computing the voltage across the capacitor (C_{p1}) and current flowing through the inductor (L) as a function of time. The energies are computed as a function of time using the energy formulas for capacitors and inductors [14]. The energies are averaged over time to obtain $|\overline{W}_m|$ and $|\overline{W}_e|$. These time-dependent energies are also added to compute the total energy storage as a function of time, which is in turn used to extract the maximum energy storage ($W_{\max}$). There are significant differences between the Q_{CONV} and $Q_{E\mathrm{MAX}}$.

TABLE I
MODEL PARAMETERS FOR 15-nH and 5-nH INDUCTORS

	Inductor 1	**Inductor 2**
L	**15.2 nH**	**4.8 nH**
R_s	**28 Ω**	**10.7 Ω**
C_{P1}	**0.26 pF**	**0.44 pF**
C_{P2}	**0.26 pF**	**0.42 pF**
R_{sub1}	**104 Ω**	**373 Ω**
R_{sub2}	**120 Ω**	**351 Ω**
# of turns	**8, Metal 2**	**6.5, Metal 1 & 2**
Inductor Trace Width & Space	**10 μm (W) 5.0 μm (S)**	**10 μm (W) 2.0 μm (S)**
Outer Area	**310 x 310 μm^2**	**140 x 140μm^2**

Fig. 1(b) shows the average stored magnetic and electrical energies as well as the maximum energy storage for the 15-nH inductor as a function of frequency. At low frequencies, the total energy is dominated by the magnetic energy and Q factors estimated using $[-\mathrm{Im}(y_{11})/\mathrm{Re}(y_{11})]\,(Q_{CONV})$ are close to $Q_{E\mathrm{MAX}}$. However, as frequency is increased, the electrical energy storage increases, and the difference between the average stored magnetic and electrical energies decreases, which in turn increases the difference between Q_{CONV} and $Q_{E\mathrm{MAX}}$. As a matter of fact, since the self-resonant frequency of an inductor occurs near a frequency where the difference between the average stored magnetic and electrical energy is zero, or where the imaginary part of y_{11} is equal to zero, from (2), Q factors extracted using $-\mathrm{Im}(y_{11})/\mathrm{Re}(y_{11})$ become zero near the self-resonant frequency. This result, of course, is physically unreasonable. The quality factor should not be zero at the self-resonant frequency.

Lastly, it was stated earlier that accurately estimating $Q_{E\mathrm{MAX}}$ is difficult. This is due to problems with representing parasitic as well as inductive element(s) in the model, and extracting their values from measurements. Related difficulties include the distributed nature of the model elements and their frequency dependence. The equivalent circuit model at best is rough. These problems are exacerbated by the fact that the model parameters are extracted using y-parameters which depend on the difference between the average magnetic and electrical energies rather than on the maximum total energy storage. Hence, the use of an extracted equivalent model for computation of the maximum energy storage (and $Q_{E\mathrm{MAX}}$) is prone to errors.

III. EFFECTIVE QUALITY FACTORS OF INTEGRATED INDUCTORS

Before going further, an examination of reasons behind interests for quality factors is in order. For microwave and RF applications, their importance arises from the fact that, for matching networks, Q factors are related to loss, while for bandpass filters, they are related to the 3-dB bandwidths [15]. Quality factors are also related to the phase or frequency stability, and phase noise (through the bandwidth) of oscillators using parallel RLC circuits. The frequency stability factor of an oscillator (S_F) is

$$S_F = -\omega_o \frac{d\phi}{d\omega}\bigg|_{\omega=\omega_o} = -\omega_o \frac{d}{d\omega}\left[\text{atan}\left(\frac{\text{Im}(y_{11})}{\text{Re}(y_{11})}\right)\right]\bigg|_{\omega=\omega_o} \tag{3}$$

where ω_o is the resonant frequency [16]. Actual parameters of interest are the loss, bandwidths of resonant circuits, and frequency stability factors of oscillators.

Since the bandwidth and frequency stability factor are defined only at the resonant frequency, measured y_{11} data can be used to extract them at the self-resonant frequency of an inductor. These, however, are not typically of great interest because using an inductor in circuits requires adding parasitic or intentional capacitances and resistances in parallel with the inductor. Once a capacitor is added, the resonant frequency is lowered from the inductor self-resonant frequency, and the bandwidth and stability factor are also changed. Unfortunately, these cannot be obtained directly from the measured y_{11} data. An obvious way to estimate these is to fabricate a set of test structures consisting of an inductor and a capacitor connected in parallel with varying capacitance values, and to characterize the structures at their respective resonant frequencies. This of course requires large numbers of test structures and measurements.

Luckily, the fabrication and measurements of the test structures are not necessary. Instead, a bandwidth and a frequency stability factor at a resonant frequency different from the inductor self-resonant frequency can be obtained by numerically adding a capacitor ($C_{num}\omega j$) in parallel to the measured y_{11} data, and by computing the parameters at the resonant frequency of the resulting RLC circuit. The resulting bandwidth and frequency stability factor represent the smallest bandwidth and the highest stability factor which can be achieved using a given inductor at the resonant frequency, since in real circuits, adding a capacitor cannot be accomplished without adding resistance or increasing the loss. To compute these parameters over a range of frequencies of interest, the resonant frequency must be swept, which can be accomplished by sweeping the added capacitance value.

The quality factor is still a useful figure of merit that provides means for quickly estimating circuit performance. The 3-dB bandwidth ($\Delta\omega$) of simple RLC circuits [15], and the frequency (phase) stability factor of oscillators [16] using simple parallel RLC circuits, are related to the quality factor by

$$Q = \frac{\omega_o}{\Delta\omega} = Q_{BW} \tag{4}$$

$$Q = \frac{S_F}{2} = Q_{PS}. \tag{5}$$

As seen in the equivalent circuit model of Fig. 1(a), the RLC circuit resulting from numerically adding a capacitor to the equivalent circuit is not a simple RLC circuit. Despite this, using (4) and (5), effective quality factors Q_{BW} (based on the bandwidth) and Q_{PS} (based on the phase stability) can be defined. It should be emphasized that these are effective values. In addition, as mentioned earlier, integrated inductors fabricated in silicon technologies in general have low Q factors, and this makes the validity of the bandwidth-based Q factor definition questionable [15]. However, as figures of merit for estimating and comparing usefulness of inductors, these effective values are still quite useful. Because of their close ties to circuit applications, these effective quality factors should also be more useful and relevant figures of merit than Q_{CONV}. An important feature of the effective Q extractions is that they do not require an equivalent circuit model. This eliminates the modeling and model parameter extraction errors. The extraction routines are implemented in Matlab [17], and the routines are around 30 lines long and straightforward.

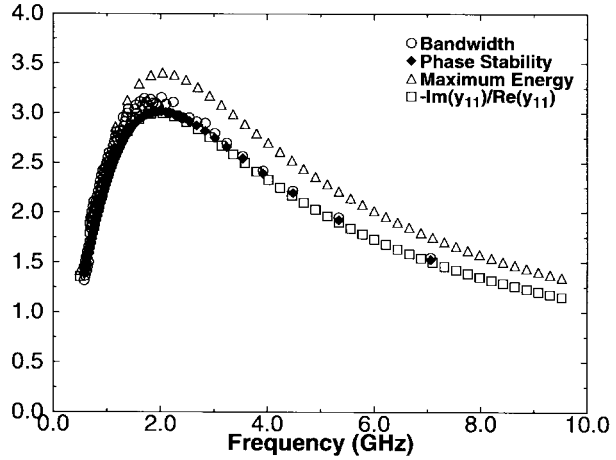
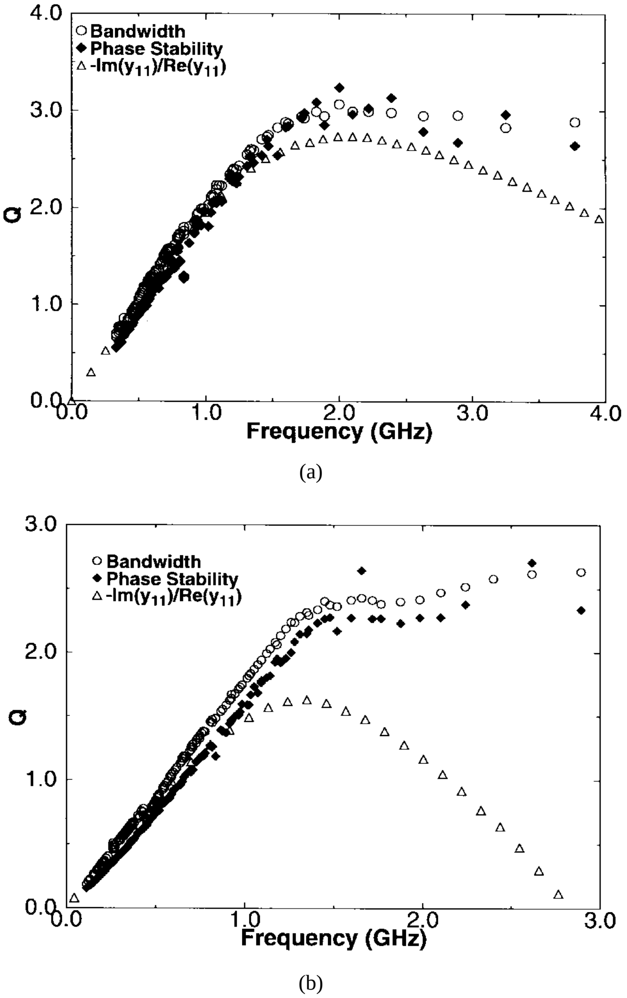


Fig. 2. Quality factor versus frequency plots of a 5-nH inductor.

IV. DISCUSSION

In order to compare Q_{BW} and Q_{PS} to $Q_{CONV}[-\text{Im}(y_{11})/\text{Re}(y_{11})]$ as well as to $Q_{E\text{MAX}}$ (based on the maximum energy storage consideration), Q_{BW} and Q_{PS} are extracted using the equivalent circuit model parameters given in Table I. Figs. 1(a) and 2 show these quality factors for the 15-nH and 5-nH inductor, respectively. In Fig. 1(a), Q_{CONV} deviates significantly from $Q_{E\text{MAX}}$, while Q_{CONV} deviates relatively little (10–15%) for the 5-nH inductor in Fig. 2. For the 15-nH inductor, Q_{CONV} also deviates significantly from Q_{BW} and Q_{PS}. Q_{BW} and Q_{PS} differ from $Q_{E\text{MAX}}$ by 10–35% between 1.5 and 2.5 GHz, which is significantly less than the 40–80% deviation of Q_{CONV}. The smaller differences among Q_{CONV}, Q_{BW}, Q_{PS}, and $Q_{E\text{MAX}}$ for the 5-nH inductor are due to the fact that the average magnetic energy storage is substantially larger than the electrical energy storage in the examined frequency range. There also exist some differences between Q_{BW} and Q_{PS}, although they are small. Figs. 1(a) and 2 clearly illustrate that Q_{BW} and Q_{PS} better estimate $Q_{E\text{MAX}}$, although significant differences can still exist. For designing filters, tuned amplifiers, and oscillators, Q_{BW} and Q_{PS} should be completely adequate.

Fig. 3(a) and (b) shows plots of Q_{CONV}, Q_{BW}, and Q_{PS} for 2.6- and 12-nH inductors. These quality factors are extracted directly from measured S-parameters rather than using extracted model parameters. For both inductors, at low frequencies, quality factors based on different techniques are approximately the same. However, at high frequencies, Q_{BW} and Q_{PS} are consistently higher than Q_{CONV}. Q_{CONV} can become zero or even negative, while Q_{BW} and Q_{PS} remain



(a)

(b)

Fig. 3. (a) and (b) Plots of Q_{CONV}, Q_{BW}, and Q_{PS} versus frequency for 2.6- and 12-nH inductors. Quality factors are extracted directly from measured S-parameter data.

well above the zero all the way up to the self-resonant frequencies of the inductors. This, of course, is physically more reasonable. For instance, the 12-nH inductor in Fig. 3(b) has a self-resonant frequency of 2.9 GHz. At this frequency, Q_{BW}, Q_{PS}, and Q_{CONV} are 2.6, 2.4, and 0, respectively. In addition, frequency dependence of Q_{BW} and Q_{PS} at high frequencies is less than that of Q_{CONV}. These data indicate that inductors remain useful all the way up to their resonant frequencies, which is contrary to that suggested by quality factors extracted using the conventional technique. The Q_{PS} data are more scattered because their extraction involves computations of derivatives.

V. SUMMARY

Through an examination of uses for quality factors, new methods for estimating quality factors of integrated inductors for RF and microwave applications are proposed. These methods involve a numerical addition of varying capacitance in parallel with measured y_{11} data, and extractions of the 3-dB bandwidths and frequency stability factors. Implementations of extraction routines require simple numerical manipulations of measured S-parameters. At low frequencies, quality factors from the new and conventional $[-\mathrm{Im}(y_{11})/\mathrm{Re}(y_{11})]$ methods are approximately the same. On the other hand, at moderate to high frequencies, the conventional method can significantly underestimate quality factors which is caused by the fact that the imaginary part of y_{11} is related to the difference between the average magnetic and electrical energy storage rather than the maximum total energy storage. The conventional Q definition can lead to improper use and optimization of integrated inductors.

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